

Probability

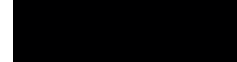
1. Properties of Probability

(1) Terminology

- \emptyset denotes the null or empty set;
- $A \subseteq B$ means A is a subset of B;
- $A \cup B$ is the union of A and B;
- $A \cap B$ is the intersection of A and B;
- A^c is the complement of A (i.e., all elements in S that are not in A).
- Mutually exclusive and Exhaustive events:

A_1, A_2, \dots, A_k are mutually exclusive events means that $A_i \cap A_j = \emptyset, i \neq j$; that is, A_1, A_2, \dots, A_k are disjoint sets;

A_1, A_2, \dots, A_k are exhaustive events means that $A_1 \cup A_2 \cup \dots \cup A_k = S$



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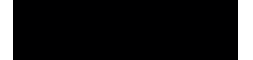
(3) Venn diagrams

(4) Definition

Probability is a real-valued set function P that assigns, to each event A in the sample space S , a number $P(A)$, called the probability of the event A , such that the following properties are satisfied:

- a) $P(A) \geq 0$;
- b) $P(S) = 1$;
- c) if A_1, A_2, A_3, \dots are events and $A_i \cap A_j = \emptyset$, $i \neq j$, then

$$P(A_1 \cup A_2 \cup \dots \cup A_k) = P(A_1) + P(A_2) + \dots + P(A_k)$$



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- d) For each event A, $P(A) \leq 1$.
- e) If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- f) If A, B, and C are any three events, then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

(1) Definition

The conditional probability of an event A, given that event B has occurred, is

defined by $P(A|B) = \frac{P(A \cap B)}{P(B)}$, $P(B) > 0$.



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If A is an event, then A is the union of m mutually exclusive events, namely,

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup \dots \cup (B_m \cap A)$$

