Subject: Calculas
Created by: Rishita Kar
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Rates of Change and Derivatives

1. Average Rate of Change: The average rate of change is given by the change in the “y” values over the change in the “x” values.

   For $y = f(x)$, the average rate of change from $x = a$ to $x = a + h$ is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \quad h \neq 0$$

2. Instantaneous Rate of Change: The instantaneous rate of change is given by the slope of a function ( ) evaluated at a single point = .

   For $y = f(x)$, the instantaneous rate of change at $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad \text{if the limit exists}$$

3. Derivative: The derivative of a <u>function</u> represents an infinitesimal change in the function with respect to one of its variables. It is also represented by the slope of the tangent like at a particular point for the function curve. The "simple" derivative of a function f with respect to a variable x is — , also denoted as $f'(x)$.

   Here are some ways to find the derivative of a function:

   a. Using the Definition of the Derivative

   For $y = f(x)$, we define the derivative of f at x, denoted $f'(x)$,by

$$f'(x) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

If $f'(x)$ exists for each x in the open interval (a,b),then f is said to be differentiable over (a,b).

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NOTE: For more formulas, refer to the Differentiation and Integration Formulas handout.

Here are some examples where the derivative ass the slope of the tangent can be applied:

Find an equation of the tangent line to the curve at the given point.

$$y = 2x^3 - x^2 + 2 \ ; \ (1,3)$$

First, find the derivative: $y' = 6x^2 - 2x$

->remember that derivative equals slope, 'm.'

Secondly, plug in the x-value from the point given, (**1**,3) into the derivative.

$$y' = 6(\ )^2 - 2(\ ) = 4$$

therefore, m=4

Thirdly, use point-slope formula to find the tangent line.

Point-slope formula: − = (−) (,) = (,)

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$$y - 3 = 4(x - 1)$$

$$y - 3 = 4x - 4$$

$$y = 4x - 1$$ --this is the tangent line.

1) Find the equation of the normal line to the curve at the given point. (1,3)

-> Normal line means perpendicular line.

The slope ('m') of a perpendicular line is the negative reciprocal, for example, if an equation has slope, $m = \frac{2}{3}$, it's perpendicular line will have slope, $m = -\frac{3}{2}$.

Since the our given equation had slope, $m = \frac{3}{1}$, it's normal line will have slope, $m = -\frac{1}{3}$.

Now, use point slope formula again using slope, m = − –.

Point-slope formula :

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− = − − + −

y-3=-1/3(x-1)

y-3=-1/3x+1/3