

Rates of Change and Derivatives

1. Average Rate of Change: The average rate of change is given by the change in the "y" values over the change in the "x" values.

For $y = f(x)$, the average rate of change from $x = a$ to $x = a + h$ is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{f(a+h) - f(a)}{h} \quad h \neq 0$$

2. Instantaneous Rate of Change: The instantaneous rate of change is given by the slope of a function () evaluated at a single point = .

For $y = f(x)$, the instantaneous rate of change at $x = a$ is

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \quad \text{if the limit exists}$$

3. Derivative: The derivative of a function represents an infinitesimal change in the function with respect to one of its variables. It is also represented by the slope of the tangent line at a particular point for the function curve. The "simple" derivative of a function f with respect to a variable x is — , also denoted as $f'(x)$.

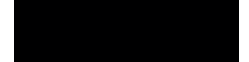
Here are some ways to find the derivative of a function:

- a. Using the Definition of the Derivative

For $y = f(x)$, we define the derivative of f at x , denoted $f'(x)$, by

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

If $f'(x)$ exists for each x in the open interval (a,b) , then f is said to be differentiable over (a,b) .



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NOTE: For more formulas, refer to the Differentiation and Integration Formulas handout.

Here are some examples where the derivative as the slope of the tangent can be applied:

Find an equation of the tangent line to the curve at the given point.

$$y = 2x^3 - x^2 + 2 ; (1,3)$$

First, find the derivative: $y' = 6x^2 - 2x$

->remember that derivative equals slope, 'm.'

Secondly, plug in the x-value from the point given, (1,3) into the derivative.

$$y' = 6(1)^2 - 2(1) = 4$$

therefore, $m=4$

Thirdly, use point-slope formula to find the tangent line

Point-slope formula: $y - y_1 = m(x - x_1)$ where $(x_1, y_1) = (1, 3)$

$$y - 3 = 4(x - 1)$$

$$y - 3 = 4x - 4$$

$$y = 4x - 1 \quad \text{--this is the tangent line.}$$

1) Find the equation of the normal line to the curve at the given point. (1,3)

-> Normal line means perpendicular line.

The slope m of a perpendicular line is the negative reciprocal for example if an equation has slope, $m = \frac{2}{3}$ its perpendicular line will have slope $m = -\frac{3}{2}$.

Since the our given equation had slope, $m = \frac{3}{1}$ its normal line will have slope $m = -\frac{1}{3}$.

Now, use point slope formula again using slope, $m = -\frac{1}{3}$.

Point-slope formula :



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$$- = - - + -$$

$$y-3=-1/3(x-1)$$

$$y-3=-1/3x+1/3$$

